MODULE TWO

This module addresses solving linear equations.

SC Academic Elementary Algebra Standards included in this module are:

EA-4.7	Carry out procedures to solve linear equations for one variable algebraically
EA-3.6	Classify a variation as either direct or inverse
EA-3.7	Carry out a procedure to solve literal equations for a specified variable.
EA-3.8	Apply proportional reasoning to solve problems.

The following solving linear equations activity can be used an introductory activity or as a review activity after Lesson #1.

WHAT IS A JIGSAW? First, each member of the home team is assigned a topic on which to become an expert. Then each student from the home teams is assigned a topic or skill. Student with the same topic/skill will meet in expert groups to discuss and master the topic/skill. All students return to their home teams and share/teach what they learned to their team members.

EXAMPLE: A pre-assessment is given to help create heterogeneous groups based on ability level. Place students in home teams of five. Within home teams, assign each member to one of the following expert groups: one step addition and subtraction equations, one step multiplication and division equations, two step equations, simplifying expressions involving distributive property and the checker.

Students move to their expert groups. While the teacher serves as facilitator, students will be given time to help each other master the assigned skill. Each group will have a practice sheet with ONLY problems related to that skill. The checker group will have problems with answers. They learn how to verify solutions using substitution (calculators will be needed). Students should also be able to explain to their home group why they performed a certain step. Students who are having difficulty solving linear equation may be assigned to the checker group in order to build confidence. A primary goal of the activity is to give each student support and assistance.

Then students will move back to their home groups. Before they begin working, the teacher will explain and model the Cycle for Solving Equations: Distributive Property First → Two Step → One Step. If the problem doesn't have one of steps of the cycle, students move on to the next step. Later, the Cycle will be Distribute Property First → Variables on Both Sides → Two Step → One Step.

Given an equation like $2x = 7$, the group facilitator will ask for the first step to solving this equation. The one step expert will share the first step, explain why and work out the problem. Then, the checker will verify.

Problems will increase in difficulty. The goal is for each "expert" to recognize where their expertise is needed. For example, given $4(2x—3) = -8$. The distributive property expert will explain how to simplify then the two step expert will give the next step then the one step expert. The checker will verify.

<u>Later:</u> The teacher can lead the class in a discussion on how to solve an equation like $3x — 4 = -5x + 6$ by asking the two step experts "What could we do to put this equation in the two step equation format?" or "What should be eliminated in order to put this equation in the two step equation format?" Then follow the cycle.

Lesson # 1
Topic: Solving Linear Equations
Standards (s): EA – 4.7

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
  - In 6th grade, students solve one step linear equations with whole number solutions and coefficients (6-3.5). In 7th grade, students solved two step equations and inequalities (7-3.4). In 8th grade, students solved multi-step equations (8-3.4)
  - In elementary algebra, students carry out procedures to solve linear equations for one variable algebraically.
  - The process of solving linear equations is foundational for student's work with solving other types of equations such as quadratic (IA-3.3) and polynomial equations (IA-4.3) for an indicated variable.

- **Taxonomy Level**
  3.1-C
  Cognitive Process Dimension: Apply
  Knowledge Dimension: Procedural Knowledge

- **Key Concepts**
  Linear equation
  Solution

II. Teaching the Lesson

In this lesson, students will solve one step, two step and multi-steps linear equations as well as verify their solutions. In addition to becoming fluent in such procedures, students build a conceptual understanding of how linear equations are structured. For example, have students verbalize that the structure of the linear equation 2x – 5 = -3 is "two times some number minus five is equal to -3." This allows students to use appropriate operations to undo the structure. Students are using their prior knowledge of algebraic expressions and engaging in algebraic reasoning to solve linear equations. Presenting a real world problem whose solution is found by solving a linear equation creates purpose for the process.
See the Resources section for an opening activity.

- **Essential learning and understanding**

It is essential for students to do the following for the attainment of this indicator:

- ○Use appropriate algebraic techniques to solve for a given variable.
- ○Understand which algebraic techniques or properties were applied in order to get the resulting equivalent linear equation.
- ○Solve linear equation involving one step, two steps, distributive property, variables on both sides, fractional coefficients, decimals and the collecting of like terms.
- ○Solve linear equations that result in one solution, no solution or infinitely many solutions.
- ○Check their solutions using an appropriate method.

- **Examples of Essential Tasks**

These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.

- ○ Solve. $-2(x-5)=3x+4$
- ○ Solve. $6x-x=-10$
- ○ Solve. $\frac{2}{3}(6x-3)=4x+1$
- ○ Solve. $3(x+2)=3x+4$

- ○ In which step did the first error occur?

$$3(2x-1)=6$$

$$Step\ 1: \quad 6x-3=6$$

$$Step\ 2: \quad 6x=3$$

$$Step\ 3: \quad x=\frac{1}{2}$$

- **Non-Essential Learning and Understanding**

It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:

- ○Solve equations involving radical notation
- ○Solve rational equations with variables in the denominator that are beyond simple proportional reasoning problems.

- **Examples of Non-Essential Tasks**

The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.

- Solve $\frac{3}{x+2} = \frac{5}{2x-1}$

- Solve $\sqrt[3]{x-2} = 5$

- **Misconceptions/Common Errors**
  - Students do not fully isolate the variable. In this example, students may stop at $6x = 9$ and incorrectly conclude that $x = 9$

$$3(2x-1) = 6$$
$$6x - 3 = 6$$
$$6x = 9$$
$$x = 9$$

  - Student may misuse the equality symbol by setting up a string of equalities such as $2x + 1 = -5 = 2x = -6 = x = -3$.

  - Students may have the misconception that the variable should always be on the left side of the equals sign. This can cause confusion for students when problems are presented in the form $-3 = x$.

- **Technology Notes**
  - When checking their solution by direct substitution, students may use a graphing utility to verify their computations.
  - Student may check their solutions by using the intersect feature on their graphing utility. For example, given $2x + 1 = -5$, students would determine where the graphs of $y = 2x + 1$ and $y = -5$ intersect.
  - One of the difficulties students may have when using a graphing utility to determine the point of intersection is setting an appropriate viewing window that clearly displays the point of intersection. Students need sufficient practice performing this skill.
  - The table of values can also be used to verify for which x value are the y values of both equations equal. The table will need to be set in order to display the appropriate values.

III. Assessing the Lesson

Assessment Guidelines: *The objective of this indicator is for the student to carry out a procedure to solve linear equations. Therefore, the primary focus of the assessment should be for students to carry out such procedures*.

- **Assessment Item Examples**

○ Solve. 3x – 1 = 4
a. x = 2
b. x = $\frac{5}{3}$
c. x = $\frac{3}{5}$
d. x = -1

○ Solve. 3t – t = 12
A. t = -4
B. t = 12
C. t = -3
D. t = 6

○ Find the solution to the equation 5p + 3 = 3p + 1.
A. 1
B. -1
C. $\frac{1}{2}$
D. $\frac{-1}{2}$

○ What is the solution for 4(x - 5) = x + 7?
A. x = 9
B. x = 4
C. x = 13
D. x = 27

○ Find the solution to the equation 4(y – 2) = 2(y + 7).
A. -12
B. $\frac{22}{6}$
C. 12
D. $\frac{-22}{6}$

IV. Resources

Activity: Many students are solving linear equations without understanding what the process is. Having students, first, solve linear equations using a table of values can deepen their conceptual understanding of the process. For example, given the linear equation 2x – 5 = -1 – x, discuss the structure of the equation. Then select values of x, create a table of values by evaluating each linear expression and analyze the table of values to

determine where the two expressions are equal. Within the table, students should show the work.

X	**2x - 5**	**1 – x**
-2	2 (-2) – 5 = -9	1 – (-2) = 3
-1	2 (-1) – 5 = -7	1 – (-1) = 2
0	2 (0) – 5 = -5	1 – (0) = 1
1	2 (1) – 5 = -3	1 – (1) = 0
2	2 (2) – 5 = -1	1 – (2) = -1

So we can write that $2x - 5 = 1 - x$ is true when $x = 1$. Put students in pairs and give them other examples to work through in this manner. Then discuss how although this process is legitimate, it could be very time-consuming so algebraic methods are used.

Lesson # 2
Topic: Solving Literal Equation
Standards (s): EA – 3.7

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
  - In 8th grade, students apply procedures to solve multistep equations (8-3.4).
  - In Elementary Algebra, students carry out a procedure to solve literal equations for a specified variable. (EA-3.7).
  - In Intermediate Algebra and Pre-calculus, students use this skill to solve equations with optimization problems. In Pre-calculus, students carry out a procedure to write a rule for the inverse of a function, if it exists (PC-2.9) and need to be able to solve for variables other than x and y.

- **Taxonomy Level**
  3.1-C
  Cognitive Process Dimension: Apply
  Knowledge Dimension: Procedural Knowledge

- **Key Concepts**
  Solving equations
  Literal equations

II. Teaching the Lesson

A literal equation is an equation that contains more than one variable. Some textbooks identify solving literal equations as solving formulas for one of its variables. In this lesson, students transfer their understanding of solving linear equations to solving literal equations. In literal equations, the letters not being solved for are treated like constants. Unlike with linear equations, the result is an algebraic expression not a numerical value. Although the process of solving literal equations is similar to solving linear equations, students may have difficulty connecting the two. Emphasizing how the two representations are different first will help students to focus on what new skills need to be added to the current process. For example, given the linear equation $2x + 5 = 4$ and the literal equation $2b + c = 4$, what are the differences? How are they similar? How would you solve for x? How would you solve for b?

- **Essential Learning and Understanding**
  It is essential for students to do the following for the attainment of *this* indicator:
  - Use inverse operations to solve literal equations for a specified variable that may involve multiple steps.

- **Examples of Essential Tasks**
  These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.

  - Solve ax = b+ cd for c.
  - P = 2l + 2w, solve for w.
  - $E = MC^2$ solve for M
  - I = PRT, solve for T
  - V = IR, solve for I
  - D = RT, solve for R

- **Non-Essential Learning and Understanding**
  It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:
  - Solve equations for a variable that requires finding roots of the equation.

- **Examples of Non-Essential Tasks**
  The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.
  - $A = \pi r^2$, solve for r.

- **Misconceptions/Common Errors**
  - Students may not isolate the variable.
  - Students may stop at an intermediate step.

- **Technology Note**
  - Solving literal equations is an essential skill for using spreadsheets for solving problems.
  - Using a spreadsheet to calculate pay with or without overtime.

III. Assessing the Lesson

Assessment Guidelines: *The objective of this indicator is for the student to carry out a procedure to solve literal equations for a specified variable. The solution may involve multiple steps.*

- **Assessment Item Examples**
  - The formula for converting Celsius to Farenheit is $F = \frac{9}{5}C + 32$.

    Solve this formula for C.

    A. $C = \frac{5}{9}F + 32$

    B. $C = \frac{5}{9}(F + 32)$

    C. $C = \frac{9}{5}F - 32$

    D. $C = \frac{5}{9}(F - 32)$

  - Solve $2x + 3y = 5x + 6$ for x.

    A. $x = \frac{5x - 3y + 6}{2}$

    B. x = 3

    C. x = y – 2

    D. $x = \frac{3x - 6}{7}$

IV. Resources

Lesson # 3
Topic: Applying Proportional Reasoning to solve problems
Standards (s): EA – 3.8

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
  - In 8th grade, students apply ratios, rates, and proportions (8-2.7) and use proportional reasoning and the properties of similar shapes to determine the length of a missing side (8-5.1).
  - In Elementary Algebra, students apply proportional reasoning to solve problems.
  - This essential skill is used in all subsequent study of mathematics.

- **Taxonomy Level**
  3.1-C
  Cognitive Process Dimension: Apply
  Knowledge Dimension: Procedural

- **Key Concepts**
  Ratio
  Proportion

II. Teaching the Lesson

In this lesson, student gain a deeper understanding of linear relationships by applying proportional reasoning to solve problems. Students are introduced to concept of direct variation in this lesson; therefore, the components of the next lesson on indicator EA – 3.6 related to direct variation should be integrated into instruction.

- **Essential Learning and Understanding**
  It is essential for students to do the following for the attainment of this indicator:
  - Use proportional reasoning to solve problems.

- **Examples of Essential Tasks**
  These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.

- The variables x and y vary directly and y = 35 when x = 7. Find the value of y when x = 9.
- The distance traveled by a car moving at a constant speed varies directly with the length of time it travels. If the car travels 172 miles in 4 hours, how many miles will it travel in 9 hours?
- An equation that reflects the relationship between x and y is x/y = 40. Find the value of x when y = 5.
- A statue is to be constructed using a 10:1 (height of statue:height of person) scale. If the person to be depicted is 76 inches tall, how tall should the statue be built?

- **Non-Essential Learning and Understanding**
  It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:
  - Determine the constant of proportionality for contextual, real-world problems.

- **Examples of Non-Essential Tasks**
  The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.
  - The table below shows heights of people and their arm span (distance between fingertips with arms extended perpendicular to body). Approximate the constant of proportionality and write an equation that summarizes the relationship between these two measurements, if one exists.

Height (H) in inches	Armspan (A) in inches
64	68
71	73
62	60
68	72
65	63

- **Misconceptions/Common Errors**
  None noted

- **Technology Note**
  Use technology where appropriate.

III. Assessing the Lesson

Assessment Guidelines: *The objective of this indicator is for the student to use proportional reasoning to solve problems.*

- **Assessment Item Examples**

  - The variables x and y vary directly and y = 20 when x = 4. Find the value of y when x = 6.
    A. 24
    B. 30
    C. 80
    D. 25

  - The circumference (c) of a circle varies directly with the radius (r) of the circle. When the radius 3, the circumference is 6π. What is the circumference (c) when the radius is 4?
    A. 8π
    B. 8
    C. 4π
    D. 4

IV. Resources

Lesson # 4
Topic: Direct and inverse variation
Standards (s): EA – 3.6

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
  - In 7th grade, students classify relationships as either directly proportional, inversely proportional, or nonproportional. In 8th Grade, students apply ratios, rates, and proportions (8-2.7) and use proportional reasoning and the properties of similar shapes to determine the length of a missing side (8-5.1).
  - In Elementary Algebra, students will classify a variation as either direct or inverse.
  - In Geometry, students use scale factors to solve problems involving scale drawings and models (G-2.6). They also apply congruence and similarity relationships among triangles to solve problems (G-3.8) and apply congruence and similarity relationships among shapes (including quadrilaterals and polygons) to solve problems (G-4.6).

- **Taxonomy Level**
  2.3-B
  Cognitive Process Dimension: Understand
  Knowledge Dimension: Conceptual Knowledge

- **Key Concepts**
  Direct variation
  Inverse variation

II. Teaching the Lesson

Students have prior knowledge of direct and inverse variation from 7th grade (7-3.7). In the previous lesson, students made connections between proportional reasoning and direct variation. In this lesson, students solidify their conceptual understanding of these concepts. Distance-time relationships are an effective way of illustrating inverse relationships. These relationships should be examined graphically, tabularly, verbally and algebraically to ensure conceptual understanding.

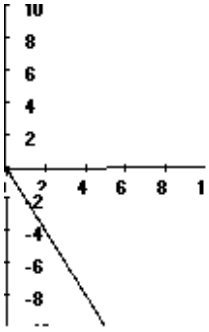
- **Essential Learning and Understanding**
  It is essential for students to do the following for the attainment of this indicator:
  - Understand the definition of direct variation
  - Understand the definition of inverse variation
  - Classify a variation as direct or inverse.

- **Examples of Essential Tasks**
  These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.

  Classify each of the following as direct variation or inverse variation.

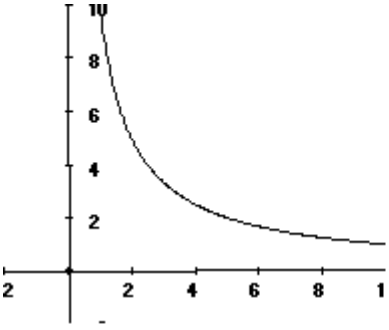
  - The circumference C of a circle and its diameter d are related by the equation C = π d.
    Answer: direct variation
  - XY = 20
    Answer: inverse variation

  - Answer: direct variation



  - Y = 4x
    Answer: direct variation

  - Answer: inverse variation



- A car is traveling at a constant speed of 50 miles/hour. The distance that the car travels is related to the time by the equation d = 50 t, where t is in hours.
  Answer: direct variation

- **Non-Essential Learning and Understanding**
  It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:
  Write the equation for a direct variation or inverse variation

- **Examples of Non-Essential Tasks**
  The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.

  Y varies directly with x and y = 12 when x = 3. Write an equation that relates x and y.

- **Misconceptions/Common Errors**
  Students may reverse the definitions. They may also have difficulty classifying some forms of equations.

- **Technology Note**
  Use technology where appropriate.

III. Assessing the Lesson

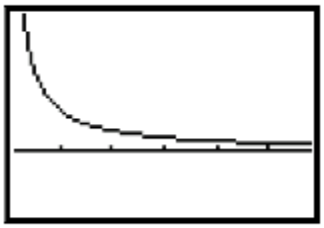
Assessment Guidelines: *The objective of this indicator is for the student to classify a variation as either direct or inverse. In addition to classifying variations, students should be able to exemplify, explain, or compare variations.*

- **Assessment Item Example**

  - The perimeter of a square is given in terms of the length of a side of the square as shown in the formula $P = 4s$. How does P vary with s?
    A. Inversely
    B. Directly
    C. Jointly
    D. There is no variation

  - Which of the functions shows inverse variation?
    A. y = 5x
    B. y = x + 3
    C. xy = 2
    D. y = x
  - Which of the following is an example of a direct variation?

A. $P = 2l + 2w$
B. $A = \pi r^2$
C. $A = lw$
D. $C = 2\pi r$



- The graph above shows
  A. Direct variation
  B. Inverse variation
  C. Joint variation
  D. A graph of all 3 types of variation

IV. Resources